C20 SOLUTION

The characteristic polynomial of $\boldsymbol{B}$ is :

El polinomio caracteristico de $\boldsymbol{B}$ es:

$$\begin{aligned} \text{pB(x)} &= \det\,(\boldsymbol{B} - \boldsymbol{x}\boldsymbol{I}_2) \\ &= \begin{vmatrix} -12-x & 30 \\ -5 & 13-x \end{vmatrix} \\ &= (-12-\boldsymbol{x})(13-\boldsymbol{x}) - (30)(-5) \\ &= \boldsymbol{x}^2 - \boldsymbol{x} - 6 \\ &= (\boldsymbol{x}-3)(\boldsymbol{x}+2) \end{aligned}$$

From this we find eigenvalues $\lambda = 3,\ -2$ with algebraic multiplicities $\alpha_B(3)$=1 y $\alpha_B(-2)$=1. For eigenvectors and geometric multiplicities, we study the null spaces of $\boldsymbol{B} - \lambda I_2$

De aqui encontramos los valores propios $\lambda = 3, -2$ con multiplicidad algebraica igual a $\alpha_B(3)$=1 y $\alpha_B(-2)$=1. Para los vectores propios y multiplicidad geometrica, estudiamos los espacios nulos de B $- \lambda \boldsymbol{I}_2$)(⟨acronymref|theorem|EMNS⟩)

Para $\lambda = 3$ B - $3\lambda\boldsymbol{I}_2$= $\begin{pmatrix} -15 & 30 \\ -5 & 10 \end{pmatrix} \longrightarrow \begin{pmatrix} \boxed{1} & 2 \\ 0 & 0 \end{pmatrix}$

$$\varepsilon_B(3) = \mathcal{N}(\text{B} - 3\boldsymbol{I}_2) = \left\langle \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \right\rangle$$

Para $\lambda = -2$ B + $2\lambda\boldsymbol{I}_2$=$\begin{pmatrix} -10 & 30 \\ -5 & 15 \end{pmatrix} \longrightarrow \begin{pmatrix} \boxed{1} & -3 \\ 0 & 0 \end{pmatrix}$

$$\varepsilon_B(-2) = \mathcal{N}(\text{B}+2\boldsymbol{I}_2) = \left\langle \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\} \right\rangle$$

Each eigenspace has dimension one, so we have geometric multiplicities $\gamma_B(3)$=1 and $\gamma_B(-2)$=1

Cada espacio propio tiene dimension uno, entonces tenemos multiplicidad geometrica $\gamma_B(3)$=1 y $\gamma_B(-2)$=1

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